Class: S.Y.BSc IT　　　　　　　　　　　　　　**Subject: Applied Mathematics**

14/10/19.

[Time:$2\frac{1}{2}$ Hours]

[Marks:75]

NOTE:

> ➤ **All questions are compulsory.**
> ➤ **Figures to the right indicate marks.**
> ➤ **Use of calculator is allowed.**

Q.1. Attempt any three from the following: [5X3=15]

A. If A and B are symmetric matrices then show that AxB is symmetric if and only if AxB and BxA exist and if AxB=BxA.

B. Reduce the matrix to normal form and find it's rank
where $A = \begin{bmatrix} 1 & -1 & 3 & 6 \\ 1 & 3 & -3 & -4 \\ 5 & 3 & 3 & 11 \end{bmatrix}$

C. Verify cayley-hamilton thoerm for the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

D. Express (1+i)(2+3i)/(4-i) in the form of a+ib and also find it's amplitude and modulus.

E. Simplify $(\cos3x - i\sin3x)^{2/3} / (\cos2x - i\sin2x)^{7/2}$.

F. Using De-Movie's theorem prove that $(1+i)^8 + (1-i)^8 = 32$.

Q.2. Attempt any three from the following:　　　　　　　　　　　　[5X3=15]

A. Solve x dy/dx + y^2/x = y.

B. Solve $(-2x^2 + y)$ dx + $(-x + x^{2y})$ dy = 0.

C. Solve (p - 2x)(p - y) = 0.

D. Solve: d^3y/dx^3 - 3 d^2y/dx^2 + 3 dy/dx - y= e^x + xe^x.

E. Solve: y = xp + 1/p.

F. Solve: $(D + 4D)y = \sin 2x$.

Q.3. Attempt any three from the following: [5X3=15]

A. Find Laplace transform of f(t) = t.
B. State and prove convolution theorem of Laplace transform.
C. Evaluate L[sin at].
D. Find inverse Laplace transform of 1/(s^2 +6s+5).
E. Find inverse Laplace transform, by using convolution theorem, 1/(s+1)(s^2 +1).
F. Find Laplace transform of: f(t)=1 when 0<t<a and = -1 when a<t<2a.

Q.4. Attempt any three from the following: [5X3=15]

A. Evaluate $\int_0^1 \int_0^y xy\, e^{-x^2}\, dx\, dy$.

B. Change to polar $\int_0^{4a} \int_{\frac{y^2}{4a}}^{y} (x^2 - y^2)/(x^2 + y^2)\, dy\, dx$.

C. Evaluate $\int_{-1}^{1} dz \int_0^z dx \int_{x-z}^{x+z}(x + y + z)dy$.

D. Evaluate $\iint xy(x + y)dx\, dy$ over the area between curve y=x^2 and line y=x.

E. Calculate the volume of the solid bounded by the following surfaces: z=0, $x^2+y^2=1$, x+y+z=3

F. Evaluate $\int_0^2 \int_0^x \int_0^{2x+2y} e^{(x+y+z)}\, dx\, dy\, dz$.

Q.5. Attempt any three from the following: [5X3=15]

A. Prove that beta(m,n) ={gamma (m) gamma(n)}/(gamma(m+n)) relation between beta and gamma function.
B. Prove that beta(1/4) beta(3/4) = pi.root(2).
C. Evaluate $\int_1^{2a} x\sqrt{2ax - x^2}\, dx$ in terms of beta function and find it's value.
D. Find: d/dx [erf(x) + erfc(ax)].
E. Prove that: d/dx [erf(ax^n)] = (2an/$\sqrt[p]{x^{n-1}}$. $e^{-a^2x^{2n}}$.
F. Verify the rule of differentiation under the integral sign for $\int_1^{a^2} tan^{-1}ax\, dx$. Evaluate the integral.

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