

2Hrs 30 min

75 Marks

Please check whether you have got the right question paper.

- Note :**
- All questions are compulsory. (Subject to Internal Choice)
 - Figures to the right indicate full marks.
 - Use of non-programmable calculator is allowed and mobile phones are not allowed.
 - Normal distribution table is printed on the last page for reference.
 - Support your answers with diagrams / illustrations, wherever necessary.
 - Graph papers will be supplied on request.

Q1.A) Multiple choice questions (Attempt Any 8)**(8)**

- i) In linear programming, unbounded solution means _____
 a) Infeasible solution b) Degenerate solution c) Infinite solutions d) Unique solution
- ii) If $M + N - 1 =$ Number of allocations in transportation, it means _____
 (Where 'M' is number of rows and 'N' is number of columns)
 a) There is no degeneracy b) Problem is unbalanced
 c) Problem is degenerate d) Solution is optimal
- iii) Floats for critical activities will be always _____
 a) One b) Zero c) Highest d) Same as duration of the activity
- iv) The total time required to complete all the jobs in a job sequencing problem is known as _____
 a) Idle time b) Processing time c) Elapsed time d) Processing order
- v) In linear programming, _____ represents mathematical equation of the limitations imposed by the problem.
 a) Objective function b) Decision variable c) Redundancy d) Constraints
- vi) If in an assignment problem, number of rows is not equal to number of columns then _____
 a) Problem is degenerate b) Problem is unbalanced
 c) It is a maximization problem d) Optimal solution is not possible
- vii) The maximum time in which an activity will be completed assuming all possible delays and postponements is termed as _____
 a) Optimistic time b) Most likely time c) Pessimistic time d) Expected time
- viii) The various alternatives or courses of actions available to each player in a game are called as _____
 a) Saddle points b) Strategies c) Pay off d) 'n' player game
- ix) In simplex, a maximization problem is optimal when all Delta J, i.e. $C_j - Z_j$ values are _____
 a) Either zero or positive b) Either zero or negative
 c) Only positive d) Only negative
- x) Which of the following considers difference between two least costs for each row and column while finding initial basic feasible solution in transportation?
 a) North west corner rule b) Least cost method
 c) Vogel's Approximation method d) Row minima method

Q1.B) True or false (Attempt Any 7)

(7)

- i) Probability of a project completing in its expected time (T_e) will be always 100%.
- ii) If saddle point is available in a game, it is called as pure strategy game.
- iii) Slack represents unutilized resources.
- iv) If in a transportation problem, number of rows is not equal to number of columns, then the problem is unbalanced.
- v) If we introduce an unnecessary dummy activity, the error is termed as redundancy.
- vi) Job sequencing problems are solved to ensure that, both, the total time to complete all jobs and idle time of each machine are maximum.
- vii) When more than one optimal solution is possible in a linear programming problem, it is termed as 'unique solution'.
- viii) Regret matrix is made to convert a maximization problem into minimization problem in assignment.
- ix) Critical path method (CPM) considers the three time estimates: most likely, optimistic and pessimistic time estimates.
- x) In solving a job sequencing problem, it is assumed that all jobs require the same sequence of operations.

Q2. A)

(8)

A company produces 2 products A and B. x_1 and x_2 are the quantities manufactured of Products A and B respectively. The following objective function along with constraints is given to you:

$\text{Max } Z = 8x_1 + 16x_2$

Subject to constraints:

$x_1 + x_2 \leq 200$

$x_2 \leq 125$

$x_1 + 2x_2 \leq 300$

$x_1 \geq 0; x_2 \geq 0$

Find how many units of Product A and Product B should be produced by the company so that the profit is maximized. Is it a case of multiple optimal solutions? Use **graphical method** to solve the LPP.

Q2.B) You are given the per unit cost of transporting goods from 3 factories to 4 customers.

The 3 factories A, B and C have capacity to supply 500, 300 and 200 units respectively.

The 4 customers P, Q, R and S require 180, 150, 350 and 320 units respectively.

| Factory \ Customers | P | Q | R | S |
|---------------------|----|----|----|----|
| A | 12 | 10 | 12 | 13 |
| B | 7 | 11 | 8 | 14 |
| C | 6 | 16 | 11 | 7 |

(i) You are required to find the Initial Basic Feasible Solution using Vogel's Approximation Method. (5)

(ii) Find the total cost of transportation schedule obtained using VAM. (2)

OR

Q2.C) You are given the following details for a project consisting of 8 activities:

| ACTIVITY | NODE | DURATION (days) |
|----------|-------|-----------------|
| A | 1 - 2 | 4 |
| B | 1 - 3 | 6 |
| C | 1 - 5 | 13 |
| D | 2 - 3 | 5 |
| E | 2 - 4 | 20 |
| F | 4 - 6 | 10 |
| G | 3 - 6 | 6 |
| H | 5 - 6 | 16 |

- (i) Draw the network diagram and identify the critical path. (3)
- (ii) Find earliest start time, earliest finish time, latest start time and latest finish time for each activity. (4)
- (iii) Find free float for activity B. (1)

Q2. D) There are 6 jobs to be performed in a factory and each would go through 2 machines A and B in the order AB. The processing time (in hours) is given for each job in each machine.

| Job | Machine (A) | Machine (B) |
|-----|-------------|-------------|
| I | 7 | 3 |
| II | 4 | 8 |
| III | 2 | 6 |
| IV | 5 | 6 |
| V | 9 | 4 |
| VI | 8 | 1 |

- (i) Determine the sequence of performing the jobs that would minimize the total time of completing all the jobs. (2)
- (ii) Find total elapsed time. (3)
- (iii) Find idle time for both the machines. (2)

Q3.A) Six jobs are to be processed in three machines A, B and C in the order A-B-C. You are given time for each job in each machine.

| JOB | Time in Machine (A) - hours | Time in Machine (B) - hours | Time in Machine (C) - hours |
|-----|-----------------------------|-----------------------------|-----------------------------|
| I | 12 | 3 | 7 |
| II | 8 | 4 | 10 |
| III | 7 | 2 | 9 |
| IV | 11 | 5 | 6 |
| V | 10 | 2 | 11 |
| VI | 5 | 4 | 4 |

- (i) Find the sequence that minimizes the total elapsed time required to complete the jobs. (2)
- (ii) Calculate the total elapsed time (3)
- (iii) Find idle time on Machine A, Machine B and Machine C. (3)

Q3.B) You are given the following details for a project with 8 activities:

| Activity | Node | Optimistic time (days) | Most likely time (days) | Pessimistic time (days) |
|----------|-------|------------------------|-------------------------|-------------------------|
| A | 1 – 2 | 4 | 6 | 8 |
| B | 2 – 3 | 5 | 7 | 15 |
| C | 2 – 4 | 4 | 8 | 12 |
| D | 3 – 5 | 10 | 18 | 26 |
| E | 4 – 6 | 8 | 9 | 16 |
| F | 5 – 7 | 4 | 8 | 12 |
| G | 6 – 7 | 1 | 2 | 3 |
| H | 7 – 8 | 6 | 7 | 8 |

- (i) Draw the network diagram. (3)
- (ii) Find the expected time of project completion along with standard deviation. (2)
- (iii) What is the probability of the project completing in 55 days? (2)

OR

Q3. C) You are given information about the cost (in Rs. Thousands) of performing different jobs by different persons. P1 cannot perform J3. P3 cannot perform J4.

| | | JOB | | | | |
|----------------------------|----|-----|-----|-----|-----|-----|
| P E R S O N | | J 1 | J 2 | J 3 | J 4 | J 5 |
| | P1 | 27 | 18 | X | 20 | 21 |
| | P2 | 31 | 24 | 21 | 12 | 17 |
| | P3 | 20 | 17 | 20 | X | 16 |
| | P4 | 22 | 28 | 20 | 16 | 27 |

- (i) Obtain optimal assignment and find cost of such assignment. (7)
- (ii) Is it a case of alternative optimal solution? (1)

Q3.D) Two firms, Lacko textiles and Rayon textiles have 3 strategies each to select from. The 3 strategies are no advertisement, using moderate advertising and using heavy advertising. You are given the pay off matrix from view point of Lacko textiles, showing its market share under several combinations of strategies:

pay-off in Rs.10,000/-

Rayon textiles

| | | | | |
|-----------------------|------------------|-------------|---------------|-----------------|
| | | No advt (I) | Mod advt (II) | Heavy advt (II) |
| Lacko textiles | No advt (I) | 50 | 40 | 28 |
| | Mod advt (II) | 70 | 50 | 45 |
| | Heavy advt (III) | 75 | 52 | 50 |

- (i) Find the saddle point and value of game. (2)
- (ii) Comment on the strategy to be selected by both the companies. (5)

Q4. A) You are given a solution for a transportation cost problem. Figures in each cell represent per unit transportation cost. Figures in circle within each cell represent number of units allocated for transportation. X, Y and Z are the 3 factories and A, B, C and D are the 4 customers.

| | A | B | C | D | Supply |
|--------|-----------|-----------|-----------|----------|--------|
| X | 13 | 200 7 | 19 | 0 | 200 |
| Y | 17 | 120 18 | 15 | 380 7 | 500 |
| Z | 180 11 | 22 | 100 14 | 20 5 | 300 |
| Demand | 180 | 320 | 100 | 400 | |

- (i) You are required to check the above solution for optimality. (3)
- (ii) If it is not optimal, use modified distribution method to obtain optimal solution. (3)
- (iii) Find optimal transportation cost. (2)

B) You are given the following information for a project with 8 activities:

| Node | Normal Duration (days) | Crash cost per day (Rs) | Maximum possible crash time |
|-------|------------------------|-------------------------|-----------------------------|
| 1 – 2 | 6 | 80 | 2 |
| 1 – 3 | 8 | 90 | 4 |
| 1 – 4 | 5 | 30 | 2 |
| 2 – 4 | 3 | - | 0 |
| 2 – 5 | 5 | 40 | 2 |
| 3 – 6 | 12 | 200 | 4 |
| 4 – 6 | 8 | 50 | 3 |
| 5 – 6 | 6 | - | 0 |

The cost of completing the eight activities in normal time is Rs.6,500.

Indirect cost is Rs.160 per day. The contract includes a penalty of Rs.100 per day for every day of delay more than 17 days.

- (i) Draw the network diagram and find critical path. (3)
- (ii) Crash the project duration to find the total cost of completing the project in 17 days (4)

OR

Q4.C) A company produces 2 products x_1 and x_2 using three resources S_1 , S_2 and S_3 . Product x_1 gives profit of Rs.30 per unit and product x_2 gives profit of Rs.40 per unit. The 3 resources S_1 , S_2 and S_3 are available to the extent of 200 units, 600 units and 500 units respectively. The following objective function and constraints are given to you:

$$\text{Max } Z = 30x_1 + 40x_2$$

Subject to constraints:

$$x_1 + 2x_2 \leq 200$$

$$8x_1 + 5x_2 \leq 600$$

$$3x_1 + 4x_2 \leq 500$$

$$x_1 \geq 0; x_2 \geq 0$$

You are given the following simplex solution to the above problem:

| | | $C_j \rightarrow$ | 30 | 40 | 0 | 0 | 0 |
|----|-------|-------------------|-------|-------|---------|-------|-------|
| C | X | B | x_1 | x_2 | S_1 | S_2 | S_3 |
| 40 | x_2 | 100 | 1/2 | 1 | 1/2 | 0 | 0 |
| 0 | S_2 | 100 | 11/2 | 0 | (-) 5/2 | 1 | 0 |
| 0 | S_3 | 100 | 1 | 0 | (-) 2 | 0 | 1 |
| | | $Z_j \rightarrow$ | 20 | 40 | 20 | 0 | 0 |

A) With reference to the above table answer the following :

- i) Check if the above solution is optimal or not. (2)
- ii) If it is not optimal, find optimal solution. (5)

B) With reference to the optimal simplex table in the above problem obtained by you, answer the following:

- iii) Find the optimal product mix and optimal profit (2)
- iv) Which resources are scarce and which are unutilized? (2)
- v) Is it a case of alternative solution? Justify your answer (2)
- vi) What are the shadow prices of the resources? Justify. (2)

Q5 A) Explain the concepts: Total float, Free float, Independent float and Interfering float. (8)

B) Discuss any 5 areas where techniques of operations research can be applied. (7)

OR

C) Answer any 3 of the following : (15)

- i) Explain the terms: Redundant constraint and infeasibility in linear programming
- ii) What do you mean by alternative optimal solution in transportation? How do you identify alternative solution in a transportation problem? Further what is the procedure to find that alternative solution?
- iii) Explain time cost trade off in project crashing.
- iv) Discuss the significance of theory of games. Briefly discuss the terms: Players and Pay off.
- v) Distinguish between PERT and CPM.

NORMAL DISTRIBUTION TABLE

Area Under the Standard Normal Distribution

| Z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|-----|---------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.0 | 0.0000 | 0.0040 | 0.0080 | 0.0120 | 0.0160 | 0.0199 | 0.0239 | 0.0279 | 0.0319 | 0.0359 |
| 0.1 | 0.0398 | 0.0438 | 0.0478 | 0.0517 | 0.0557 | 0.0596 | 0.0636 | 0.0675 | 0.0714 | 0.0753 |
| 0.2 | 0.0793 | 0.0832 | 0.0871 | 0.0910 | 0.0948 | 0.0987 | 0.1026 | 0.1064 | 0.1103 | 0.1141 |
| 0.3 | 0.1179 | 0.1217 | 0.1255 | 0.1293 | 0.1331 | 0.1368 | 0.1406 | 0.1443 | 0.1480 | 0.1517 |
| 0.4 | 0.1554 | 0.1591 | 0.1628 | 0.1664 | 0.1700 | 0.1736 | 0.1772 | 0.1808 | 0.1844 | 0.1879 |
| 0.5 | 0.1915 | 0.1950 | 0.1985 | 0.2019 | 0.2054 | 0.2088 | 0.2123 | 0.2157 | 0.2190 | 0.2224 |
| 0.6 | 0.2257 | 0.2291 | 0.2324 | 0.2357 | 0.2389 | 0.2422 | 0.2454 | 0.2486 | 0.2518 | 0.2549 |
| 0.7 | 0.2580 | 0.2612 | 0.2642 | 0.2673 | 0.2705 | 0.2734 | 0.2764 | 0.2797 | 0.2823 | 0.2852 |
| 0.8 | 0.2881 | 0.2910 | 0.2939 | 0.2967 | 0.2995 | 0.3023 | 0.3051 | 0.3078 | 0.3106 | 0.3133 |
| 0.9 | 0.3159 | 0.3186 | 0.3212 | 0.3238 | 0.3264 | 0.3289 | 0.3315 | 0.3340 | 0.3365 | 0.3389 |
| 1.0 | 0.3413 | 0.3438 | 0.3461 | 0.3485 | 0.3508 | 0.3531 | 0.3554 | 0.3577 | 0.3599 | 0.3621 |
| 1.1 | 0.3643 | 0.3665 | 0.3686 | 0.3708 | 0.3729 | 0.3749 | 0.3770 | 0.3790 | 0.3810 | 0.3830 |
| 1.2 | 0.3849 | 0.3869 | 0.3888 | 0.3907 | 0.3925 | 0.3944 | 0.3962 | 0.3980 | 0.3997 | 0.4015 |
| 1.3 | 0.4032 | 0.4049 | 0.4066 | 0.4082 | 0.4099 | 0.4115 | 0.4131 | 0.4147 | 0.4162 | 0.4177 |
| 1.4 | 0.4192 | 0.4207 | 0.4222 | 0.4236 | 0.4251 | 0.4265 | 0.4279 | 0.4292 | 0.4306 | 0.4319 |
| 1.5 | 0.4332 | 0.4345 | 0.4357 | 0.4370 | 0.4382 | 0.4394 | 0.4406 | 0.4418 | 0.4429 | 0.4441 |
| 1.6 | 0.4452 | 0.4463 | 0.4474 | 0.4484 | 0.4495 | 0.4505 | 0.4515 | 0.4525 | 0.4535 | 0.4545 |
| 1.7 | 0.4554 | 0.4464 | 0.5473 | 0.4582 | 0.4591 | 0.4599 | 0.4608 | 0.4616 | 0.4625 | 0.4633 |
| 1.8 | 0.4641 | 0.4649 | 0.4656 | 0.4664 | 0.4671 | 0.4678 | 0.4686 | 0.4693 | 0.4699 | 0.4706 |
| 1.9 | 0.4713 | 0.4719 | 0.4726 | 0.4732 | 0.4738 | 0.4744 | 0.4750 | 0.4756 | 0.4761 | 0.4767 |
| 2.0 | 0.4772 | 0.4778 | 0.4783 | 0.4788 | 0.4793 | 0.4798 | 0.4803 | 0.4808 | 0.4812 | 0.4817 |
| 2.1 | 0.4821 | 0.4826 | 0.4830 | 0.4938 | 0.4838 | 0.4842 | 0.4846 | 0.4850 | 0.4854 | 0.4857 |
| 2.2 | 0.4861 | 0.4846 | 0.4868 | 0.4871 | 0.4875 | 0.4878 | 0.4881 | 0.4884 | 0.4887 | 0.4890 |
| 2.3 | 0.7893 | 0.4896 | 0.4898 | 0.4901 | 0.4904 | 0.4906 | 0.4909 | 0.4911 | 0.4913 | 0.4916 |
| 2.4 | 0.4918 | 0.4820 | 0.4922 | 0.4925 | 0.4927 | 0.4931 | 0.4931 | 0.4932 | 0.4934 | 0.4936 |
| 2.5 | 0.4938 | 0.4940 | 0.4941 | 0.4943 | 0.4945 | 0.4946 | 0.4948 | 0.4949 | 0.4951 | 0.4952 |
| 2.6 | 0.4953 | 0.4955 | 0.4956 | 0.4958 | 0.4959 | 0.4960 | 0.4961 | 0.4962 | 0.4963 | 0.4964 |
| 2.7 | 0.4965 | 0.4966 | 0.4967 | 0.4968 | 0.4969 | 0.4970 | 0.4971 | 0.4972 | 0.4973 | 0.4974 |
| 2.8 | 0.4974 | 0.4975 | 0.4976 | 0.4977 | 0.4977 | 0.4978 | 0.4979 | 0.4979 | 0.4980 | 0.4981 |
| 2.9 | 0.4981 | 0.4982 | 0.4982 | 0.4983 | 0.4984 | 0.4984 | 0.4985 | 0.4985 | 0.4988 | 0.4986 |
| 3.0 | 0.49865 | 0.4987 | 0.4987 | 0.4988 | 0.4988 | 0.4989 | 0.4989 | 0.4989 | 0.4990 | 0.4996 |
| 4.0 | 0.49968 | | | | | | | | | |
