F.Y.B.Sc.IT- Sem I- A.T.K.T. Exam- Mar'2023

Dt:- 21/3/23

F.Y.B.Sc.(IT) SEM-I

DISCRETE MATHEMATICS

Total Marks :75

NOTE:

(i) All the questions are compulsory.
(ii) All the questions carry equal marks.
(iii) Calculator is allowed.

Q-1 Attempt Any Three: **[15]**

[1] Of 21 typists in an office, 5 use all manual typewriters(M), electronic typewriters (E) and word processors(W); 9 use E and W; 7 use M and W; 6 use E and M; but no one uses M only. If the same number of typists use electronic as use word processors, then how many use word processor only?

[2] Use mathematical induction to prove that $1 + 2 + 2^2 + \ldots\ldots\ldots\ldots 2^n = 2^{n+1} - 1$ for all integers $n \geq 0$.

[3] Do as directed:

(i) Define Difference of a set.

(ii) Let U = {1,2,3.....10}, X = {1,2,3,4,5}, Y = {y/y = 2x, x∈ X}, Z = {2, 7}
Enumerate: $Y \cup Z$.

(iii) Let U = {1,2,3,4,5}, C= {1,3}. Also, A and B are nonempty sets. $A \cup B = U$, $A \cap B = \emptyset$ and B = {1}. Find A.

[4] A = {1,2,3,4}, B = {x, y, z}. Let R be a relation from set A to B given by,
R = {(1, x), (2, z), (3, x), (3, y), (3, z)} Find (a) R^{-1} (b) Domain and Range of R.

[5] Define the cartesian product of sets. Let A = {1,2,3} and B = {a, b} find A × B & B × A.

[6] Let A = {1, 2, 3, 4, 6}. Let "R" be a relation on set A such that xRy iff "x completely divides y", Write R and from R write the matrix of R.

Q-2 Attempt Any Three: **[15]**

[1] Let X = {a, b, c} and Y = {1, 2, 3, 4}.

(i) Define a function f: X→ Y by arrow diagram.

(ii) Find range of f.

(iii) Find the domain of f.

(iv) Find f(a), f(b) and f(c).

[2] Let f: R→ R be defined by $f(x) = x^3 + 5$. Show that f is one-to-one and onto.

[3] Let f: R→ R be defined by f(x) = 3x - 7. It is given that f is one -to- one and onto. Find a formula that defines the inverse function f^{-1}.

[4] From a pack of a well shuffled 52 cards, four cards are drawn successively with replacement, find the probability that all 4 cards are spades?

[5] There are 3 doctors, 4 engineers, 2 statisticians and 1 economist. A committee of 4 among them is to be formed. Find the probability that committee consists of at least one doctor.

[6] Find the expected number of heads that can be obtained in a throw of two coins.

Q-3 Attempt Any Three: **[15]**

[1] (i) There are 5 gentlemen and 4 ladies. In how many ways a group of 2 men and 2 ladies can be formed?

(ii)How many 4-digit numbers can be formed out of the digits 1,2,3,5,7,8,9 if no digit is Repeated in any number?

[2] A box contains 7 red, 6 white and 4 blue balls. How many ways 3 balls can be selected so that (i) none is red (ii) one is of each colour.

[3] If nCr =56 and nPr = 336, find r and n.

[4] Write Pigeon-Hole Principle. Show that in any set of 11 integers, there are at least two integers whose difference is divisible by 10.

[5] Write Extended Pigeon-Hole Principle. Show that if there are 100 students in a class then at least there are four of them whose first name begins with the same alphabet.

[6] Define Recurrence Relation. Find the first five terms of the following recursively defined sequence.$t_0 = -1, t_1 = 1, t_k = t_{k-1} + 2.t_{k-2}$, for all integers k ≥ 2.

Q-4 Attempt Any Three: **[15]**

[1] Define:(i) Handshaking Theorem (ii) Degree of a vertex (iii) Complete graph

[2]

(i) Determine the number of edges in a graph with 6 nodes, 2 of degree 4 and 4 of degree 2.

(ii)How many nodes are necessary to construct a graph with exactly 6 edges in which each is of degree 2?

[3] Draw a connected regular graph of degree 0,1 and 2.

[4] Let V = {1, 2, 3, 4}, E = {e_1, e_2, e_3, e_4, e_5, e_6} and edges e_1, e_2, e_3, e_4, e_5, e_6 are associated with pair of vertices (1, 2), (4, 3), (1, 3), (2, 4), (1, 2), (1, 2), (3 3) respectively. Represent the above information by drawing a graph G = (V, E).

[5] In the following either draw a graph with specified...or explain why no such graph exists.

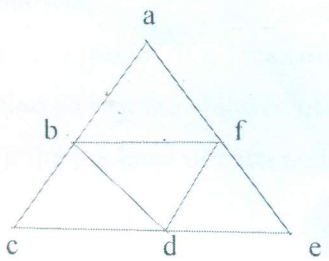
(i) Graph with 5 vertices of degrees 1, 2, 3, 3 and 5.

(ii) Graph with 4 vertices of degrees 1, 2, 3 and 3.

[6] For the following graph find

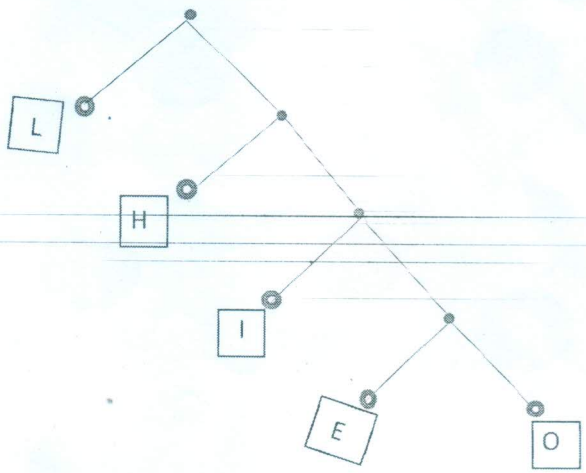
(i)Degree of each vertex and verify handshaking Theorem

(ii)Write Adjacency matrix



Q-5 Attempt Any Three: **[15]**

[1] Consider the prefix code tree in the following figure:



Decode the following code strings. (a) 0111001111(b) 10111101110

[2] The following statements are whether True or False:

(i) If all the vertices of graph have even degree, then the graph is Eulerian.

(ii)The outdegree of a vertex v in a directed graph is the number of edges ending or terminating at vertex v.

(iii) A graph in which all vertices of equal degree is called complete graph.

(iv) A vertex with degree one is called Pendant vertex.

(v) Degree of any vertex of a simple graph with 'n' vertices cannot exceed (n-1).

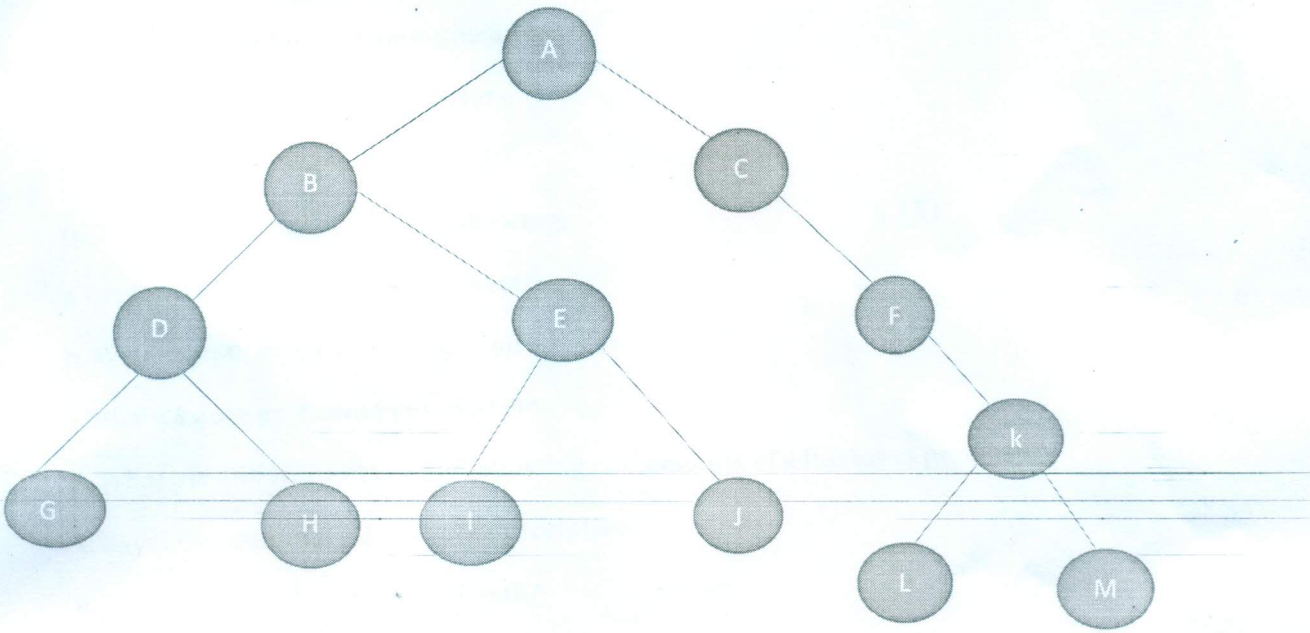
[3] Considering the following table draw Code Tree and find which are the prefix codes.

Characters	A	B	C	D	E
Code- 1	1	00	01	001	011
Code- 2	1	10	100	1000	-

[4] Define the following:

(i) Binary Tree (ii) Height of a Tree (iii) Binary search Tree

[5] For the tree find (i) List the children of each node(ii)List the siblings(iii)Find the depth of each node(iv)Find the level of each node.



[6]

(i) Show that there does not exist a simple graph with 8 vertices and 29 edges.

(ii) In a connected planar graph, there are 15 vertices, each of degree 2. Find the number of regions.

X.....X.....X.....X