

All questions are compulsory with internal choice.

Figure to the right indicates marks.

Use of calculator is allowed.

Q 1. Attempt any two from the following:

[5x2 = 10]

- a) If the matrix A is a skew hermitian matrix. Find  $iA$  and comment on your answer, where

$$A = \begin{bmatrix} 1 & 1 - 2i \\ -1 - 2i & 0 \end{bmatrix}.$$

- b) Show that inverse of unitary matrices is unitary.  
c) Reduce the following matrix to the normal form and also find its rank

$$A = \begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & 3 & 4 \end{bmatrix}$$

- d) Find inverse of the following matrix by using adjoint method

$$A = \begin{bmatrix} 4 & 5 \\ 0 & 1 \end{bmatrix}.$$

Q 2. Attempt any two from the following:

[5x2 = 10]

- a) Check whether the vectors  $(2, 1, 4)$ ,  $(0, 7, 3)$  and  $(2, 1, 6)$  are linearly dependent. If so, find the relation between them.  
b) Find the solution of the following system of equation:  
 $4x - 2y + 6z = 8$ ,  $x + y - 3z = -1$ ,  $15x - 3y + 9z = 21$ .  
c) Find an inner product  $a \cdot b$  and  $b \cdot a$  where  $a = 3i - 8j + 2k$ ,  $b = i + 7j - 2k$ .  
d) Find the eigenvalues of matrix  $A = \begin{bmatrix} 8 & 4 \\ 3 & -2 \end{bmatrix}$ .

Q 3. Attempt any two from the following:

[5x2 = 10]

- a) The position vector of a moving particle at a time  $t$  is  $R = t^2i - t^3j + t^4k$ . Find the tangential and normal components of its acceleration at  $t = 1$ .  
b) A particle moves on the curve  $x = 2t^2$ ,  $y = t^2 - 4t$ ,  $z = 3t - 5$ , where  $t$  is the time. Find the velocity and acceleration at time  $t = 1$ .  
c) Find a unit vector normal to the surface  $x^3 + y^3 + 3xyz = 3$  at the point  $(1, 2, -1)$ .  
d) Find the volume of the parallelepiped whose edges are represented by the vectors  $A = -i + 2j - 4k$ ,  $B = -2i + 5j + k$ ,  $C = 3i + j + 2k$ .

Q 4. Attempt any two from the following:

[5x2 = 10]

- a) Solve  $\frac{dy}{dx} = e^{2x-y}$ .  
b) Solve  $(e^y + 1) \cos x dx + e^y \sin x dy = 0$ .  
c) Solve  $(2x^3 + 3y) dx + (3x + y - 1) dy = 0$   
d) Solve  $\frac{dy}{dx} = y$ .

Q 5. Attempt any two from the following:

[5x2 = 10]

- If  $y = ax + b\sqrt{x}$  then prove that  $2x^2y_2 - xy_1 + y = 0$ .
- Solve  $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 4y = 0$ .
- Solve  $(D^2 + 2D + 3)y = x - x^2$ .
- Solve  $(D^2 + 4D + 4)y = x^2$ .

Q 6. Attempt any two from the following:

[5x2 = 10]

- Verify Cauchy MVT for  $f(x) = \sqrt{x}$  and  $g(x) = \frac{1}{\sqrt{x}}$ , in  $[a, b]$ .
- Verify Roll's theorem for the function  $f(x) = (x^2 - 1)(x - 2)$  in  $[-1, 2]$ .
- Verify LMVT for  $f(x) = (x - 1)(x - 2)(x - 3)$  in  $[0, 4]$ .
- Verify Cauchy MVT for  $f(x) = x^2$  and  $g(x) = x$  in  $[a, b]$ .

Q 7. Attempt any three from the following:

[5x3 = 15]

- Find the eigen values of the matrix  $A = \begin{bmatrix} -2 & 3 \\ 5 & -2 \end{bmatrix}$ .
- Solve  $\log\left(\frac{dy}{dx}\right) = ax + by$ .
- If  $R = A \sin \omega t + B \cos \omega t$ , where  $A, B$  are constant vectors and  $\omega$  is a scalar then show that  $R \times \frac{dR}{dt} = -\omega(A \times B)$ .
- Solve  $(2x - y + 1) dx + (2y - x - 1) dy = 0$ .
- Verify LMVT for  $f(x) = 2x^2 - 7x + 10$  in  $[2, 0]$ .
- Evaluate  $\text{curl} [e^{xyz}(I + J + K)]$ .

\*\*\*\*\*