| 2021 | IX | 27 | 1030 | **J-582** | (E) |

# MATHEMATICS & STATISTICS (88)
## (COMMERCE)

Time : 3 Hrs.    (12 Pages)    Max. Marks : 80

**General Instructions :**

(i)   All questions are compulsory.

(ii)   There are 6 questions divided into two sections.

(iii)   Write answers of Section-I and Section-II in the same answer book.

(iv)   Use of logarithmic tables is allowed. Use of calculator is not allowed.

(v)   For L.P.P. graph paper is not necessary. Only rough sketch of graph is expected.

(vi)   Start answer to each question on a new page.

(vii)   For each multiple choice type of question, it is mandatory to write the correct answer along with its alphabet eg. (a) ................./ (b) ................/ (c) ..................../ (d) ............... No mark(s) shall be given if "*ONLY*" the correct answer is written. Only the first attempt will be considered for evaluation.

## SECTION - I

**Q. 1.** **(A)** Select and write the correct answer of the following multiple choice type of questions (1 mark each) :    **[12]** (6)

(i)   $\int \dfrac{1}{x^2-9}\,dx = $ ........

(a)   $\log\left|\dfrac{x-3}{x+3}\right|+c$

(b)   $\dfrac{1}{3}\log\left|\dfrac{x-3}{x+3}\right|+c$

(c)   $\dfrac{1}{6}\log\left|\dfrac{x-3}{x+3}\right|+c$

(d)   $\dfrac{1}{9}\log\left|\dfrac{x-3}{x+3}\right|+c$

| 0 | 5 | 8 | 2 |

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(ii) The area of the region bounded by the curve $y = 2x + 3$, the X-axis and lines $x = 0$ and $x = 3$ is ........

   (a) 18 sq. units        (b) 9 sq. units

   (c) 27 sq. units        (d) $\dfrac{21}{4}$ sq. units

(iii) If $\displaystyle\int_0^a 5x^4\,dx = 32$, $a \in \mathbb{R}$, then $a =$ ..............

   (a) 2              (b) 1

   (c) $\dfrac{2}{5}$          (d) 10

(iv) The conditional statement $p \rightarrow q$ is equivalent to ......

   (a) $p \vee \sim q$       (b) $\sim p \vee q$

   (c) $\sim p \wedge q$       (d) $q \rightarrow p$

(v) The solution of the differential equation $\dfrac{dy}{dx} = 1$ is .....

   (a) $y + x = c$       (b) $xy = c$

   (c) $x^2 + y^2 = c$       (d) $y = x + c$

(vi) If $y = Ae^{3x} + Be^{-3x}$, is the general solution of the differential equation, then the differential equation is .....

   (a) $\dfrac{d^2y}{dx^2} = 9y$       (b) $\dfrac{d^2y}{dx^2} = 3y$

   (c) $\dfrac{d^2y}{dx^2} = -3y$       (d) $\dfrac{d^2y}{dx^2} = -9y$

**(B)** State whether the following statements are true or false (1 mark each) :          (3)

(i) The product of two non-zero matrices cannot be a zero matrix.

(ii) $\displaystyle\int e^x[x^2 + 2x]\,dx = x^2 e^x + c$

(iii) The degree of differential equation $e^{\left(\frac{dy}{dx}\right)} = \frac{dy}{dx} + x^2$

is not defined.

**(C)** Fill in the following blanks (1 mark each) : (3)

    (i) If the function $f'(x) = 6x^2 - 42x + 36$ then the function $f(x)$ has minimum at $x =$ ..................... .

    (ii) $\int \dfrac{1+x}{x+e^{-x}} dx = \log\left|........+1\right| + c$,

    Where $c$ is the constant of integration.

    (iii) $\displaystyle\int_{-7}^{7} \dfrac{x^7}{x^4-9} dx =$ ...............

**Q. 2.** **(A)** Attempt any TWO of the following (3 marks each) : (6) **[14]**

    (i) Write negation of each of the following statements :

        (a) $\forall n \in N; \ n+1 > 0$

        (b) $\sqrt{5}$ is an irrational number

        (c) $p \vee \sim q$

    (ii) If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$, find K so that $A^2 - 8A - KI = 0$, where

    I is unit matrix of order 2 and $0$ is null matrix of order 2.

    (iii) Evaluate : $\displaystyle\int_{2}^{3} \dfrac{x}{(x+2)(x+3)} dx$

**(B)** Attempt any TWO of the following (4 marks each) : (8)

    (i) Find the values of $x$, such that $f(x)$ is increasing function, where

$$f(x) = 2x^3 - 15x^2 + 36x + 1$$

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(ii) Evaluate : $\int x^2 e^{4x} dx$

(iii) Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$$

**Q. 3.** **(A)** Attempt any TWO of the following (3 marks each ):     (6) **[14]**

   (i) Find $\dfrac{dy}{dx}$, if $y = (\log x)^x + x^{\log x}$

   (ii) Find the area of the region bounded by the parabola
   $y^2 = 4x$ and the line $x = 3$.

   (iii) Solve the differential equation $x\dfrac{dy}{dx} + 2y = x^2 \log x$

**(B)** Attempt any ONE of the following :     (4)

   (i) If $x^7 y^9 = (x+y)^{16}$, then show that $\dfrac{dy}{dx} = \dfrac{y}{x}$

   (ii) Prepare the truth table for the statement pattern
   $(p \wedge \sim q) \leftrightarrow (q \rightarrow p)$. Also interpret the result.

**(C)** Attempt any ONE of the following questions (Activity) :     (4)

   (i) The rate of growth of bacteria is proportional to the
   number present. If initially, there were 1000 bacteria
   and the number doubles in 1 hour; complete the
   following activity to find the number of bacteria after
   $\dfrac{5}{2}$ hours (Given $\sqrt{2} = 1.414$)

   Solution : Let N be the number of bacteria present at
   time '$t$'. Since growth of bacteria is proportional to
   the number present, the differential equation can be
   written as

| 0 | 5 | 8 | 2 |
|---|---|---|---|

$$\frac{dN}{dt} = K \boxed{\phantom{xxx}};$$

Where K is constant of proportionality.

$$\log N = \boxed{\phantom{xxx}} + c \dots\dots\dots(1)$$

(i)  When $t = 0$ ;  $N = 1000$
from equation (1)
$\log 1000 = 0 + c$
$\therefore c = \log 1000$

(ii)  When $t = 1$ ;  $N = 2000$
from equation (I)
$$K = \boxed{\phantom{xxx}}$$

(iii)  When $t = \dfrac{5}{2}$ hours

$$N = \boxed{\phantom{xxx}}$$

(ii)  If the demand function is $D = 50 - 3P - P^2$, where P is the unit price. Complete the following activity to find the elasticity of demand at  (a) $P = 5$,  (b) $P = 2$.
Solution : The demand function is $D = 50 - 3P - P^2$
Elasticity of demand is,

$$\eta = \frac{-P}{D} \cdot \frac{dD}{dP}$$

(a)  When $P = 5$, $\eta = \boxed{\phantom{xxx}}$

$\therefore$ Demand is $\boxed{\phantom{xxx}}$ for $P = 5$.

(b)  When $P = 2$,

$$\eta = \boxed{\phantom{xxx}}$$

$\therefore$ Demand is $\boxed{\phantom{xxx}}$ for $P = 2$.

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# SECTION - II

**Q. 4.** **(A)** Select and write the correct answer of the following multiple choice type of questions (1 mark each) : [12] (6)

(i) The car worth ₹ 10,00,000 insured for ₹ 7,00,000 is damaged to the extent of ₹ 5,00,000 in an accident, then the amount of compensation that can be claimed under the policy is ₹ .....

(a) 3,50,000     (b) 7,00,000

(c) 2,50,000     (d) 5,00,000

(ii) The date on which the period of the bill expires is called ..............

(a) Legal due date     (b) Grace date

(c) Nominal due date     (d) Date of drawing

(iii) If $P_{0_1}(L) = 90$  $P_{0_1}(P) = 40$ then $P_{0_1}(F) = $..........

(a) 3600     (b) 60

(c) 65     (d) 90

(iv) The following trend line equation was developed for annual sales from 1984 to 1990 with 1984 as base year, $y = 500 + 60x$, (in 1000 ₹). The estimated sales for 1984 (in 1000 ₹) is ............

(a) 500     (b) 560

(c) 1040     (d) 1100

(v) If X denotes the number of heads obtained when an unbiased coin is tossed twice then the expected value of X is ...........

(a) 1     (b) 0.5

(c) 1.5     (d) 2

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(vi) The processing times required for four jobs A, B, C and D on Machine $M_1$ are 5, 8, 10 and 7 hours, and on Machine $M_2$, it requires 7, 4, 3 and 6 hours respectively. The jobs are processed in the order $M_1$ $M_2$. The sequence that minimizes total elapsed time is ..........

(a) ABCD      (b) BCDA

(c) ADBC      (d) ABDC

**(B)** State whether the following statements are true or false :     (3)
(1 mark each)

(i) In the regression equation of Y on X, $b_{xy}$ represents slope of the line.

(ii) The quantity index number according to weighted aggregate method is given by $\dfrac{\sum q_1 w}{\sum q_0 w} \times 100$

(iii) Irregular variation is not a random component of time series.

**(C)** Fill in the blanks (1 mark each) :     (3)

(i) The present worth of a sum of ₹10,920 due six months hence at 8% p.a. Simple interest is .............. .

(ii) The Marshall-Edgeworth's Price index number is given by .............. .

(iii)

| Year | 1991 | 1992 | 1993 |
|------|------|------|------|
| Production | 2 | 3 | 4 |

From the above data, the trend value using 3 yearly moving average is .............. .

**Q. 5.** **(A)** Attempt any TWO of the following questions (3 marks each) : (6) **[14]**

(i) Find the rate of interest compounded annually, if immediate annuity of ₹ 20,000 per year amounts to ₹ 2,60,000 in 3 years.

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(ii) A company manufactures two types of chemicals A and B. Each chemical requires two types of raw materials P and Q. The table below shows number of units of P and Q required to manufacture one unit of A and one unit of B.

| Raw materials ↓ / Chemical → | A | B | Availability |
|---|---|---|---|
| P | 3 | 2 | 120 |
| Q | 2 | 5 | 160 |

The company gets profits of ₹ 350/- and ₹ 400/- by selling one unit of A and one unit of B respectively.

Formulate the problem as L.P.P. to maximize the profit.

(iii) In a partially destroyed laboratory record of an analysis of regression data, the following data are legible :

Regression equations are

$8x - 10y + 66 = 0$  and  $40x - 18y = 214$

Find on the basis of above information

(a) The mean values of $X$ and $Y$

(b) Correlation coefficient between $X$ and $Y$

(B) Attempt any TWO of the following questions :    (8)

(4 marks each)

(i) Mr. Pavan is paid a fixed weekly salary plus commission based on percentage of sales made by him. If on the sale of ₹ 68,000 and ₹ 73,000 in two successive weeks, he recieved in all ₹ 9,880 and ₹ 10,180, find his weekly salary and rate of commission paid to him.

(ii) The publisher produces 5 books on Mathematics. The books have to go through composing, printing and binding done by 3 machines A, B, C in the order ABC. The time schedule for the entire task in proper unit is as follows :

| Books → | I | II | III | IV | V |
|---|---|---|---|---|---|
| Machines ↓ | | | | | |
| Machine A | 4 | 9 | 8 | 6 | 5 |
| Machine B | 5 | 6 | 2 | 3 | 4 |
| Machine C | 8 | 10 | 6 | 7 | 11 |

Determine the total elapsed time and idle time for machines A, B and C.

(iii) If a fair coin is tossed 6 times find the probability of obtaining:
   (a) exactly 4 heads
   (b) atleast 4 heads
   (c) atmost 2 heads

**Q. 6. (A)** Attempt any TWO of the following questions (3 marks each) : (6) [14]

(i) If $X \sim P(m)$ with $m = 3$ and $e^{-3} = 0.0497$, then find
   (a) $P(X = 3)$
   (b) $P(X \geq 2)$

(ii) Solve the following L.P.P.
   Minimize $z = 7x + y$
   Subject to
   $5x + y \geq 5$
   $x + y \geq 3$
   $x \geq 0, \ y \geq 0$

(iii) The following is the p.d.f. of a r.v. X,
$$f(x) = \begin{cases} \dfrac{x}{8}; & 0 < x < 4 \\ 0; & \text{otherwise} \end{cases}$$

Find :  (a) $P(X < 1.5)$
   (b) $P(1 < X < 2)$
   (c) $P(X > 2)$

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**(B)** Attempt any ONE of the following questions : (4)

    (i) From the data of 20 pairs of observations on X and Y, following results are obtained :

$$\overline{x} = 199 \quad \overline{y} = 94, \quad \Sigma(x_i - \overline{x})^2 = 1200$$

$$\Sigma(y_i - \overline{y})^2 = 300, \quad \Sigma(x_i - \overline{x})(y_i - \overline{y}) = -250$$

    Find :

    (a) The line of regression Y on X

    (b) Estimate Y when X = 211

    (ii) Following table shows the amount of sugar production (in lakh tonnes) for year 1931 to 1942 :

| Year | Production | Year | Production |
|------|-----------|------|-----------|
| 1931 | 0 | 1937 | 4 |
| 1932 | 1 | 1938 | 5 |
| 1933 | 2 | 1939 | 6 |
| 1934 | 3 | 1940 | 7 |
| 1935 | 2 | 1941 | 4 |
| 1936 | 3 | 1942 | 8 |

    Obtain the trend values using 4 yearly centred moving averages.

**(C)** Attempt any ONE of the following questions (Activity) : (4)

    (i) In the following table, Laspeyre's and Paasche's price index numbers are equal. Complete the following activity to find $x$ :
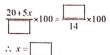
| Commodity | Base year | | Current year | |
|-----------|-----------|----------|--------------|----------|
| | Price | Quantity | Price | Quantity |
| A | 2 | 10 | 2 | 5 |
| B | 2 | 5 | $x$ | 2 |

Solution : $P_{01}(L) = P_{01}(P)$

$$\frac{\Sigma \, p_1 q_0}{\Sigma \, p_0 q_0} \times 100 = \frac{\boxed{\phantom{xx}}}{\Sigma \, p_0 q_1} \times 100$$

$$\frac{20+5x}{\boxed{\phantom{xx}}} \times 100 = \frac{\boxed{\phantom{xx}}}{14} \times 100$$

$$\therefore x = \boxed{\phantom{xx}}$$

(ii) A plant manager has four subordinates and four tasks to perform. The subordinates differ in efficiency and tasks differ in their intrinsic difficulty. Estimates of the times subordinates would take to perform tasks are given in the matrix below :

|   | I | II | III | IV |
|---|---|----|-----|----|
| A | 3 | 01 | 10 | 8 |
| B | 13 | 2 | 12 | 2 |
| C | 3 | 4 | 6 | 1 |
| D | 4 | 15 | 4 | 9 |

Complete the following activity to allocate tasks to subordinates to minimize the total time.

Solution :

Step I : Subtract the smallest element of each row from every element of that row.

|   | I | II | III | IV |
|---|---|----|-----|----|
| A | 0 | 8 | 7 | 5 |
| B | 11 | 0 | 10 | 0 |
| C | 2 | 3 | 5 | 0 |
| D | 0 | 11 | 0 | 5 |

Step II : Since all column minimums are zero, no need to subtract anything from columns.

Step III : Draw the minimum number of lines to cover all the zeros.

|   | I | II | III | IV |
|---|---|----|-----|----|
| A | 0 | 8 | 7 | 5 |
| B | 11 | 0 | 10 | 0 |
| C | 2 | 3 | 5 | 0 |
| D | 0 | 11 | 0 | 5 |

P.T.O

Since :

$$\begin{bmatrix} \text{minimum number} \\ \text{of lines} \end{bmatrix} = \begin{bmatrix} \text{Order of the} \\ \text{matrix} \end{bmatrix}$$

∴ Optimal solution has been reached

∴ Optimal assignment schedule is,

A → ▢

B → ▢

C → IV

D → ▢

∴ Total minimum time = ▢ hours.

◆ ◆ ◆