

MATHEMATICS AND STATISTICS PRELIMINARY EXAM – 2022

Time: 3 hr

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Max marks: 80

Instruction:

- 1) All questions are compulsory
- 2) Use of logarithm table is allowed. Use of calculator is not allowed
- 3) For L.P.P problem graph paper is not necessary. Only rough sketch of graph is expected.
- 4) Write answers of Section-I and Section II in the same answer book.
- 5) Start each question on a new page.
- 6) Figures to the right indicate full marks.

SECTION I

Q I.A) Select and write the most appropriate answer from the given alternatives (06)

- 1) Which one of the following is not a statement
- (a) Smoking is injurious to health
 - (b) $2 + 2 = 5$
 - (c) 2 is only even prime number
 - (d) come here

2) If $x = \frac{e^t + e^{-t}}{2}$ $y = \frac{e^t - e^{-t}}{2}$ then $\frac{dy}{dx} =$

- (a) $\frac{-y}{x}$ (b) $\frac{y}{x}$ (c) $\frac{-x}{y}$ (d) $\frac{x}{y}$

3) $\int \frac{dx}{(x-x^2)} =$

- | | |
|------------------------------|-----------------------|
| (a) $\log x - \log(1-x) + c$ | (b) $\log(1-x^2) + c$ |
| (c) $\log x + \log(1-x) + c$ | (d) $\log(x-x^2) + c$ |

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Q2.A) Attempt any two of the following (06)

(1) Write the negation of the following statements

(a) $2+3=5$ or $4+8=12$

(b) If the diagonals of a parallelogram are perpendicular then it is a Rhombus

(c) $\exists x \in \mathbb{R}$ such that $x + 3 \geq 10$

(2) Find $\frac{dy}{dx}$ if $y = (x + \frac{1}{x})^x$

(3) Find the area of the region bounded by the curve $x^2 = 25y$, $y = 1$, $y = 4$ and the y axis

Q.2.B) Attempt any two of the following (08)

(1) Solve the equation by the method of matrix inversion method

$$x+y-z = 2, \quad x-2y+z = 3, \quad 2x-y-3z = -1$$

(2) The total cost of manufacturing x articles is $c = 47x + 300x^2 - x^4$
Find the value of x for which average cost
(a) Increasing (b) decreasing

(3) Evaluate $\int \left(\frac{x}{4x^4 - 20x^2 - 3} \right) dx$

Q.3.A) Attempt any two of the following (06)

(1) Examine whether the statement pattern is a tautology or a contradiction or a contingency

$$(p \wedge \sim q) \rightarrow (\sim p \wedge \sim q)$$

(2) Find the rate of change of demand (x) of a commodity with respect to price y if $y = \frac{5x+7}{2x-13}$

(3) Evaluate $\int_0^1 \frac{1}{\sqrt{1+x} + \sqrt{x}} dx$

Q.3.B) Attempt any one of the following: (04)

(1) Find x, y, z if

$$\left\{ 3 \begin{bmatrix} 2 & 0 \\ 0 & 2 \\ 2 & 2 \end{bmatrix} - 4 \begin{bmatrix} 1 & 1 \\ -1 & 2 \\ 3 & 1 \end{bmatrix} \right\} \begin{bmatrix} 1 \\ \\ 2 \end{bmatrix} = \begin{bmatrix} x - 3 \\ y - 1 \\ 2z \end{bmatrix}$$

(2) Find the particular solution of the differential equation.

$$(x-y^2x) dx - (y+x^2y)dy = 0 \text{ when } x=2, y=0$$

Q.3.C) Attempt any one of the following: (04)

(1) Divide the number 20 in two parts such that their product is maximum

Solution: Let the first part be x

then second part =

$$\text{product} = f(x) = x(20-x)$$

$$= 20x - x^2$$

$$f'(x) = \text{} \quad f''(x) = -2$$

$$20-2x=0 \quad \therefore x=10$$

$$\text{as } f''(10) = -2 \text{ } 0$$

f(x) has maximum value at x = 10

\therefore two parts are and

(2) Form the differential equation by eliminating arbitrary constant

$$bx + ay = ab$$

$$\therefore \frac{x}{a} + \frac{y}{b} = \boxed{}$$

Diff both sides with respect to x

$$\frac{1}{a} + \frac{1}{b} \boxed{} = 0$$

$$\frac{1}{b} \frac{dy}{dx} = -\frac{1}{a}$$

$$\frac{dy}{dx} = \boxed{}$$

$$\frac{d^2y}{dx^2} = \boxed{}$$

SECTION II

Q.4.A) Select and write the most appropriate answer from the given alternatives for each subquestion (06)

(1) The cash discount is calculated on

- (a) List price (b) Cost price (c) Selling price (d) Invoice price

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(2) Price Index number by simple aggregate method

$$(a) \frac{\mathcal{E} p_1 w}{\mathcal{E} p_0 w} \times 100 \quad (b) \frac{\mathcal{E} p_0 w}{\mathcal{E} p_1 w} \times 100$$

$$(c) \frac{\mathcal{E} p_0}{\mathcal{E} p_1} \times 100 \quad (d) \frac{\mathcal{E} p_1}{\mathcal{E} p_0} \times 100$$

(3) If $E(x) = v(x)$ then x follows

- (a) Binomial Distribution
- (b) Poisson Distribution
- (c) Normal Distribution
- (d) None of the above

(4) The corner points of the feasible region are $(0,0)$, $(0,6)$, $(4.5,3)$, $(6,0)$ then the point of maximum

$$Z = 5x + 6y = 40.5 \text{ is}$$

- (a) $(0,0)$
- (b) $(0,6)$
- (c) $(4.5, 3)$
- (d) $(6,0)$

(5) In sequencing an optimal path is one that minimizes ____

- (a) Total Elapsed time
- (b) Idle time
- (c) Both (a) and (b)
- (d) In time

(6) Given p.d.f of a continuous r.v. x as $f(x) = \frac{x^2}{2}$ for $-1 < x < 2$

$f(x) = 0$ otherwise then $F(3)$ is

- (a) $\frac{1}{9}$ (b) 1 (c) $\frac{1}{2}$ (d) 0

Q.4.B) State whether the following statements are true or false (03)

- (1) byx and bxy are independent of change of origin and scale.
(2) The region represented by the inequation $x > 0$ and $y > 0$ lies in the first quadrant
(3) If random variable x takes the value 1,2,3,4,5 with equal probabilities then $E(x)$ is 3

Q.4.C) Fill in the blanks: (03)

- (1) If the installment of an annuity is due at the beginning of each period then it is called as _____
(2) The constraint that the number of second class passengers (x) is at least twice the first class passengers (y) is _____
(3) If x represents the number of heads on a single toss of unbiased coin then $\sum p(x) =$ _____

Q.5.A) Attempt any two of the following (06)

(1) The following results were obtained from records of age (x) and Blood pressure (y) of a group of 10 men.

	X	Y
Mean	50	140
Variable	150	165

And $\sum (x - \bar{x})(y - \bar{y}) = 1120$. Find the prediction of blood pressure of a man of age 40 years.

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(2) Obtain the trend values for the above data using 5 yearly moving averages

Year	1962	1963	1964	1965	1966	1967	1968	1969
Production	0	0	1	1	4	2	4	9
Year	1970	1971	1972					
Production	7	10	8					

(3) Find the value Index number using simple aggregate method

Commodity	Base Year		Current Year	
	Price	Quantity	Price	Quantity
A	30	22	40	18
B	40	16	60	12
C	10	38	15	24
D	50	12	60	16
E	20	28	25	36

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Q.5.B) Attempt any two of the following : (08)

(1) A publisher produces 5 books on mathematics. The books to go through composing, printing and binding done by 3 machines P,Q,R. The time schedule of the entire task is given

Book	A	B	C	D	E
Machine P	4	9	8	6	5
Machine Q	5	6	2	3	4
Machine R	8	10	6	7	11

Find the total elapsed time and idle time of Machine R.

(2) Maximise $z=10x + 25y$ subject to constraints $0 \leq x \leq 3$, $0 \leq y \leq 3$, $x+y \leq 5$

(3) The percentage of girls enrollment in total enrollment for years 1960 - 2005 is shown in the following table.

Year	1960	1965	1970	1975	1980	1985
Percentage	0	3	3	4	4	5
Year	1990	1995	2000	2005		
Percentage	6	8	8	10		

Fit a trend line by the method of least square

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Q.6.A) Attempt any two of the following (06)

(1) If $\sum p_0 q_0 = 110$, $\sum p_0 q_1 = 200$, $\sum p_1 q_0 = 350$, $\sum p_1 q_1 = 460$

Find Laspeyres, Paasche index number, Drobish Bowley index number.

(2) In the modification of a plant layout of a factory four new machines M_1 , M_2 , M_3 , and M_4 are to be installed in a Machine shop. There are five vacant places A, B, C, D, E available. Because of limited space machine M_2 cannot be placed at C and M_3 is not placed at A. The cost of location (in hundred rupees) is given,

	Location				
Machine	A	B	C	D	E
M_1 ,	9	11	15	10	11
M_2 ,	12	9	—	10	9
M_3 ,	—	11	14	11	7
M_4 ,	14	8	12	7	8

Find the optimum assignment schedule

(3) The p.d.f of a continuous r.v x is $f(x) = \frac{3x^2}{8}$ for $0 < x < 2$
 $f(x) = 0$ otherwise

Determine the c.d.f of x hence find

(a) $p(x < 1)$ (b) $p(1 < x < 2)$

(Q.6.B) Attempt anyone of the following

(04)

(1) The equation of two lines of regression are $3x + 2y - 26 = 0$ and $6x + y - 31 = 0$

Find

- (a) Means of x and y
- (b) Correlation coefficient between X and Y
- (c) $\text{Var}(x)$ if $\text{Var}(y) = 36$

(2) Given table indicates the p.m.f of a discrete variable X .

$x=x$	-2	-1	0	1	2	3
$P(x=x)$	$\frac{k}{2}$	k	$\frac{3k}{2}$	$3k$	k	$\frac{k}{2}$

Find (a) k (b) Find the probability distribution of x (c) Find $E(x)$

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Q.6.C) Attempt any one of the following:

(04)

(1) Complete the following activity

The difference between B.D and T.D is 8

$$\therefore \text{B.G} = \boxed{}$$

The bill is taken for a period of 6 month at a rate of 4% p.a.

B.G = simple interest on T.D

$$\boxed{} = (\text{T.D}) \left(\frac{6}{12}\right) \left(\frac{4}{100}\right)$$

$$\therefore \text{T.D} = \boxed{}$$

$$\therefore \text{B.D} = \text{B.G} + \boxed{}$$

$$\text{B.D} = 8 + 400$$

$$\text{B.D} = \text{Rs.}408$$

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(2) Complete the following activity

Let the cost of Machinery = Rs.10,00,000

Scrap Value = Rs.50,000

Effective life of Machinery = 12 years

Rate of Interest = 5% p.a.

$$\therefore i = \frac{\square}{100} = \square, \quad n = \square$$

$$A = 10,00,000 - \square = \text{Rs.}9,50,000$$

$$A = \frac{C}{i} \{(1 + i)^n - 1\}$$

$$9,50,000 = \frac{C}{\square} [1.797 - 1]$$

$$C = \frac{9,50,000 \times 0.05}{0.797} = \text{Rs.}59,584.4$$

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